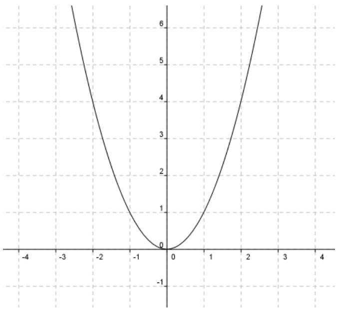
Hoofdstuk 16 Limieten

16.1 Continuïteit en differentieerbaarheid.

Opgave 1:

a.



b. $f(1)=1$

$f(1{,}9)=3{,}61$

$f(1{,}99)=3{,}9601$

$f(2{,}01)=4{,}0401$

c. voor $x=2$ wordt de noemer nul en je kunt niet door nul delen.

Opgave 2:

a. $\lim\limits_{x\to 1}\dfrac{x^2+x-2}{x-2}=\dfrac{0}{-1}=0$

b. $\lim\limits_{x\to 1}\dfrac{x^2+x-2}{x-1}=\lim\limits_{x\to 1}\dfrac{(x+2)(x-1)}{x-1}=\lim\limits_{x\to 1}x+2=3$

c. $\lim\limits_{x\to 1}\dfrac{x-1}{x^2-1}=\lim\limits_{x\to 1}\dfrac{x-1}{(x-1)(x+1)}=\lim\limits_{x\to 1}\dfrac{1}{x+1}=\dfrac{1}{2}$

d. $\lim\limits_{x\to 1}\dfrac{x-\sqrt{x}}{x^2-x}=\lim\limits_{x\to 1}\dfrac{x-\sqrt{x}}{(x-\sqrt{x})(x+\sqrt{x})}=\lim\limits_{x\to 1}\dfrac{1}{x+\sqrt{x}}=\dfrac{1}{2}$

Opgave 3:

a. $\lim\limits_{x\to 0}\dfrac{2x^2}{3x^2}=\lim\limits_{x\to 0}\dfrac{2}{3}=\dfrac{2}{3}$

b. $\lim\limits_{x\to 0}\dfrac{2x^4}{3x^3}=\lim\limits_{x\to 0}\dfrac{2x}{3}=\dfrac{0}{3}=0$

c. $\lim\limits_{x\to 0}\dfrac{2x^3}{3x^4}=\lim\limits_{x\to 0}\dfrac{2}{3x}=\text{bestaat niet}$

d. $\lim\limits_{x\to 0}\dfrac{2x^2-x}{3x}=\lim\limits_{x\to 0}\dfrac{2x-1}{3}=\dfrac{-1}{3}=-\tfrac{1}{3}$

Opgave 4:

a. $\lim\limits_{x\to 1}\dfrac{2x^2-3x+1}{1-x}=\lim\limits_{x\to 1}\dfrac{(1-x)(-2x+1)}{1-x}=\lim\limits_{x\to 1}-2x+1=-2+1=-1$

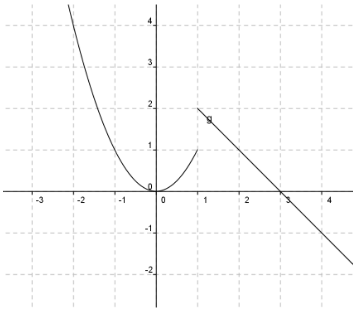
b. $\lim\limits_{x\to 4}\dfrac{x-4}{\sqrt{x}-2}=\lim\limits_{x\to 4}\dfrac{(\sqrt{x}-2)(\sqrt{x}+2)}{\sqrt{x}-2}=\lim\limits_{x\to 4}\sqrt{x}+2=2+2=4$

c. $\lim_{x \to 9} \frac{x - 3\sqrt{x}}{x^2 - 9x} = \lim_{x \to 9} \frac{x - 3\sqrt{x}}{(x - 3\sqrt{x})(x + 3\sqrt{x})} = \lim_{x \to 9} \frac{1}{x + 3\sqrt{x}} = \frac{1}{9+9} = \frac{1}{18}$

d. $\lim_{x \to 2} \frac{x^2 - 4}{x^3 - 2x^2 - 4x + 8} = \lim_{x \to 2} \frac{x^2 - 4}{(x^2 - 4)(x - 2)} = \lim_{x \to 2} \frac{1}{x - 2} =$ bestaat niet

Opgave 5:

a.



b. nee,

c. $f(1) = 1$

$-1 + p = 1$

$p = 2$

Opgave 6:

$\lim_{x \uparrow 2} f_{p,q}(x) = \lim_{x \uparrow 2} x^3 = 8$

$\lim_{x \downarrow 2} f_{p,q}(x) = \lim_{x \downarrow 2} x^2 + q = 4 + q$

$f(2) = p$

$8 = 4 + q = p$

dus $p = 8$ en $q = 4$

Opgave 7:

$\lim_{x \uparrow e} f_p(x) = \lim_{x \uparrow e} x^2 + 1 = e^2 + 1$

$\lim_{x \downarrow e} f_p(x) = \lim_{x \downarrow e} 3 + p \ln x = 3 + p$

$3 + p = e^2 + 1$

$p = e^2 - 2$

Opgave 8:

$\lim_{x \uparrow 1} f_{p,q}(x) = \lim_{x \uparrow 1} 2x - p = 2 - p$

$\lim_{x \downarrow 1} f_{p,q}(x) = \lim_{x \downarrow 1} 2^{x-1} = 2^0 = 1$

$2 - p = 1$

$p = 1$

$\lim_{x \uparrow 3} f_{p,q}(x) = \lim_{x \uparrow 3} 2^{x-1} = 4$

$\lim_{x \downarrow 3} f_{p,q}(x) = \lim_{x \downarrow 3} x^2 + q = 9 + q$

$9 + q = 4$ dus $q = -5$

Opgave 9:

a. $[-3, \rightarrow\rangle$

b. $\langle -2, \rightarrow\rangle$

c. $9 - x^2 > 0$
$-x^2 > -9$
$x^2 < 9$
$-3 < x < 3$
$\langle -3, 3\rangle$

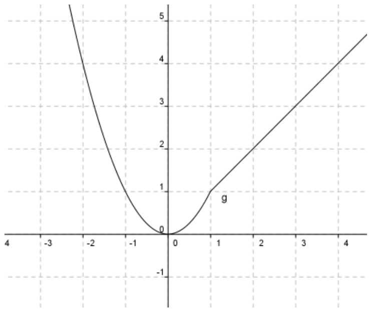
d. $\Re$

e. $\Re$

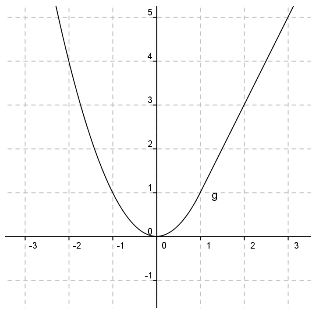
f. $2^x - 4 > 0$
$2^x > 4$
$2^x > 2^2$
$x > 2$
$\langle 2, \rightarrow\rangle$

Opgave 10:

a. $\lim_{x \uparrow 1} f_0(x) = \lim_{x \uparrow 1} x^2 = 1$
$\lim_{x \downarrow 1} f_0(x) = \lim_{x \downarrow 1} x = 1$
$\lim_{x \to 1} f_0(x) = 1 = f(1)$
dus de functie is continu in $x = 1$
ja, er zit een knik in de grafiek bij $x = 1$



b. $\lim_{x \uparrow 1} g_{-1}(x) = \lim_{x \uparrow 1} x^2 = 1$
$\lim_{x \downarrow 1} g_{-1}(x) = \lim_{x \downarrow 1} 2x - 1 = 2 - 1 = 1$
$\lim_{x \to 1} g_{-1}(x) = 1 = g(1)$
dus de functie is continu in $x = 1$
nee, er zit geen knik in de grafiek



Opgave 11:

$$f(x) = |x+3| = \begin{cases} x+3 & voor\ x \geq -3 \\ -x-3 & voor\ x < -3 \end{cases}$$

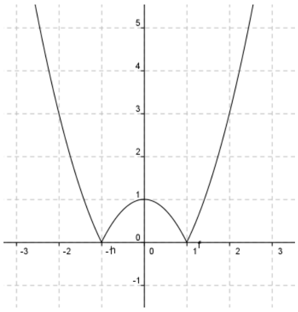
$$\lim_{h \uparrow 0} \frac{f(-3+h) - f(-3)}{h} = \lim_{h \uparrow 0} \frac{-(-3+h) - 3 - 0}{h} = \lim_{h \uparrow 0} \frac{3 - h - 3}{h} = \lim_{h \uparrow 0} \frac{-h}{h} = -1$$

$$\lim_{h \downarrow 0} \frac{f(-3+h) - f(-3)}{h} = \lim_{h \downarrow 0} \frac{-3 + h + 3 - 0}{h} = \lim_{h \downarrow 0} \frac{h}{h} = 1$$

De limieten zijn niet gelijk, dus de functie *f* is niet differentieerbaar in $x = -3$.

Opgave 12:

a.

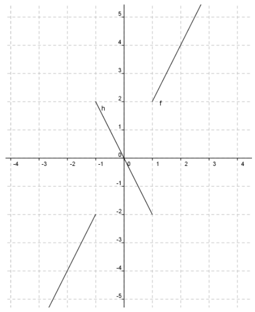


b. $|x^2-1| = \begin{cases} x^2-1 & voor\ x \le -1 \vee x \ge 1 \\ -x^2+1 & voor\ -1 < x < 1 \end{cases}$

$g(x) = \begin{cases} x^2 & voor\ x \le -1 \vee x \ge 1 \\ -x^2+2 & voor\ -1 < x < 1 \end{cases}$

$g'(x) = \begin{cases} 2x & voor\ x \le -1 \vee x \ge 1 \\ -2x & voor\ -1 < x < 1 \end{cases}$

c.



d. $x = -1$ en $x = 1$

Opgave 13:

a. $\lim_{x \uparrow 2} f(x) = \lim_{x \uparrow 2} x^2 - 1 = 4 - 1 = 3$

$\lim_{x \downarrow 2} f(x) = \lim_{x \downarrow 2} 3x - 3 = 6 - 3 = 3$

$f(2) = 3$

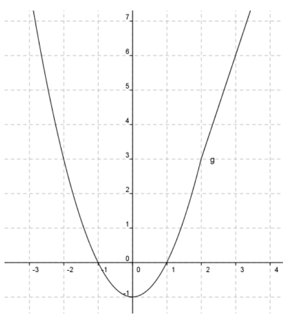
dus de functie *f* is continu in $x = 2$

b. $\lim_{h \uparrow 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \uparrow 0} \frac{(2+h)^2 - 1 - 3}{h} = \lim_{h \uparrow 0} \frac{4 + 4h + h^2 - 4}{h} = \lim_{h \uparrow 0} 4 + h = 4$

$\lim_{h \downarrow 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \downarrow 0} \frac{3(2+h) - 3 - 3}{h} = \lim_{h \downarrow 0} \frac{6 + 3h - 6}{h} = \lim_{h \downarrow 0} \frac{3h}{h} = 3$

f is niet differentieerbaar in $x = 2$

c.



Opgave 14:

a. $\lim_{x\uparrow 2} g(x) = \lim_{x\uparrow 2} x^2 - 2 = 4 - 2 = 2$

$\lim_{x\downarrow 2} g(x) = \lim_{x\downarrow 2} 3x - 3 = 6 - 3 = 3$

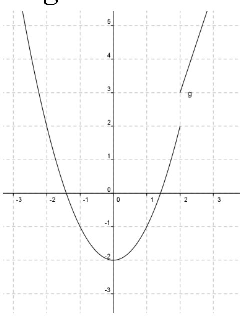
g is niet continu in $x = 2$

b. $\lim_{h\uparrow 0} \frac{g(2+h) - g(2)}{h} = \lim_{h\uparrow 0} \frac{(2+h)^2 - 2 - 2}{h} = \lim_{h\uparrow 0} \frac{4 + 4h + h^2 - 4}{h} = \lim_{h\uparrow 0} 4 + h = 4$

$\lim_{h\downarrow 0} \frac{g(2+h) - g(2)}{h} = \lim_{h\downarrow 0} \frac{3(2+h) - 3 - 2}{h} = \lim_{h\downarrow 0} \frac{6 + 3h - 5}{h} = \lim_{h\downarrow 0} \frac{1 + 3h}{h} =$ bestaat niet

dus g is niet differentieerbaar in $x = 2$

c.



Opgave 15:

a. $\lim_{x\uparrow 1} f(x) = \lim_{x\uparrow 1} -x^2 + 2x + 1 = -1 + 2 + 1 = 2$

$\lim_{x\downarrow 1} f(x) = \lim_{x\downarrow 1} x^2 - 2x + 4 = 1 - 2 + 4 = 3$

dus f is niet continu in $x = 1$

b. $f'(x) = -2x + 2$ voor $x < 1$

$f'(x) = 2x - 2$ voor $x > 1$

$\lim_{x\uparrow 1} f'(x) = \lim_{x\uparrow 1} -2x + 2 = -2 + 2 = 0$

$\lim_{x\downarrow 1} f'(x) = \lim_{x\downarrow 1} 2x - 2 = 2 - 2 = 0$

c. $\lim_{h\uparrow 0} \frac{f(1+h) - f(1)}{h} = \lim_{h\uparrow 0} \frac{-(1+h)^2 + 2(1+h) + 1 - 2}{h} = \lim_{h\uparrow 0} \frac{-1 - 2h - h^2 + 2 + 2h + 1 - 2}{h} =$

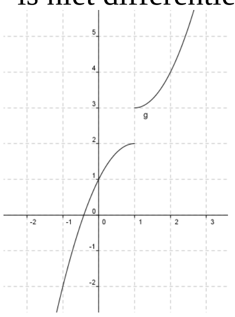
$\lim_{h\uparrow 0} \frac{-h^2}{h} = \lim_{h\uparrow 0} -h = 0$

$\lim_{h\downarrow 0} \frac{f(1+h) - f(1)}{h} = \lim_{h\downarrow 0} \frac{(1+h)^2 - 2(1+h) + 4 - 2}{h} = \lim_{h\downarrow 0} \frac{1 + 2h + h^2 - 2 - 2h + 4 - 2}{h} =$

$\lim_{h\downarrow 0} \frac{h^2 + 1}{h} =$ bestaat niet

f is niet differentieer baar in $x = 1$

d.



Opgave 16:

a. $\lim_{x \to 3} g(x) = \lim_{x \to 3} \frac{1}{2}x + 1 = 1\frac{1}{2} + 1 = 2\frac{1}{2}$

$g(3) = 2$

g is niet continu in $x = 3$

b. $g'(x) = \begin{cases} \frac{1}{2} & voor\ x \neq 3 \\ 0 & voor\ x = 3 \end{cases}$

$\lim_{x \uparrow 3} g'(x) = \lim_{x \uparrow 3} \frac{1}{2} = \frac{1}{2}$

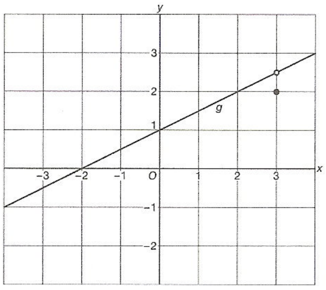
$\lim_{x \downarrow 3} g'(x) = \lim_{x \downarrow 3} \frac{1}{2} = \frac{1}{2}$

c. $\lim_{h \uparrow 0} \frac{g(3+h) - g(3)}{h} = \lim_{h \uparrow 0} \frac{\frac{1}{2}(3+h) + 1 - 2}{h} = \lim_{h \uparrow 0} \frac{1\frac{1}{2} + \frac{1}{2}h + 1 - 2}{h} = \lim_{h \uparrow 0} \frac{\frac{1}{2}h + \frac{1}{2}}{h} =$ bestaat niet

$\lim_{h \downarrow 0} \frac{g(3+h) - g(3)}{h} = \lim_{h \downarrow 0} \frac{\frac{1}{2}(3+h) + 1 - 2}{h} = \lim_{h \downarrow 0} \frac{1\frac{1}{2} + \frac{1}{2}h + 1 - 2}{h} = \lim_{h \downarrow 0} \frac{\frac{1}{2}h + \frac{1}{2}}{h} =$ bestaat niet

dus g is niet differentieerbaar in $x = 3$

d.



Opgave 17:

a. $\lim_{x \uparrow 1} f(x) = f(1) = 1$

$\lim_{x \downarrow 1} f(x) = \lim_{x \downarrow 1} 3x - 2 = 3 - 2 = 1$

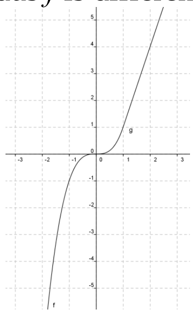
dus f is continu in $x = 1$

$\lim_{x \uparrow 1} f'(x) = \lim_{x \uparrow 1} 3x^2 = 3$

$\lim_{x \downarrow 1} f'(x) = \lim_{x \downarrow 1} 3 = 3$

dus f is differentieerbaar in $x = 1$

b.



Opgave 18:

a. $\lim_{x \uparrow 0} f(x) = \lim_{x \uparrow 0} f(0) = 2$

$\lim_{x \downarrow 0} f(x) = \lim_{x \downarrow 0} \sqrt{x + 4} = 2$

dus f is continu in $x = 0$

$\lim_{x\uparrow 0} f'(x) = \lim_{x\downarrow 0} \frac{1}{4}x + 4 = \frac{1}{4}$

$\lim_{x\downarrow 0} f'(x) = \lim_{x\downarrow 0} \frac{1}{2\sqrt{x+4}} = \frac{1}{4}$

dus *f* is differentieerbaar in $x = 0$

b. $f'(x) = \begin{cases} \frac{1}{4}x + 4 & voor\ x \leq 0 \\ \frac{1}{2\sqrt{x+4}} & voor\ x > 0 \end{cases}$

Opgave 19:

$\lim_{x\uparrow 0} f(x) = f(0) = a^2 + b$

$\lim_{x\downarrow 0} f(x) = \lim_{x\downarrow 0} x^3 + 4x + 1 = 1$

dus $a^2 + b = 1$

$\lim_{x\uparrow 0} f'(x) = \lim_{x\uparrow 0} 2(x - a) = -2a$

$\lim_{x\downarrow 0} f'(x) = \lim_{x\downarrow 0} 3x^2 + 4 = 4$

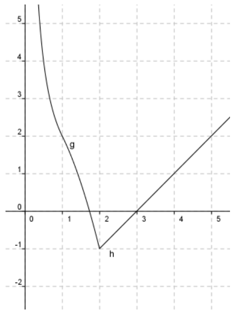
dus $-2a = 4$

$a = -2$

$b = 1 - a^2 = -3$

Opgave 20:

a.



b. $\lim_{x\uparrow 1} f(x) = \lim_{x\uparrow 1} \frac{2}{x} = 2$

$\lim_{x\downarrow 1} f(x) = f(1) = 2$

dus *f* is continu in $x = 1$

$\lim_{x\uparrow 1} f'(x) = \lim_{x\uparrow 1} -\frac{2}{x^2} = -2$

$\lim_{x\downarrow 1} f'(x) = \lim_{x\downarrow 1} -2x = -2$

dus *f* is differentieerbaar in $x = 1$

c. $\lim_{x\uparrow 2} f(x) = f(2) = -1$

$\lim_{x\downarrow 2} f(x) = \lim_{x\downarrow 2} px - 2p - 1 = -1$

dus *f* is continu in $x = 2$

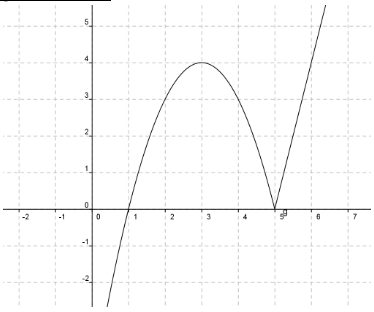
$\lim_{x\uparrow 2} f'(x) = \lim_{x\uparrow 2} -2x = -4$

$\lim_{x\downarrow 2} f'(x) = \lim_{x\downarrow 2} p = p$

dus f is differentieerbaar in $x = 2$ voor $p = -4$

Opgave 21:

a.



b. $\lim_{x \uparrow a} f(x) = \lim_{x \uparrow a} -x^2 + 6x - 5 = -a^2 + 6a - 5$

$\lim_{x \downarrow a} f(x) = f(a) = 0$

dus $-a^2 + 6a - 5 = 0$

$a^2 - 6a + 5 = 0$

$(a-1)(a-5) = 0$

$a = 1 \quad \vee \quad a = 5$

$\lim_{x \uparrow a} f'(x) = \lim_{x \uparrow a} -2x + 6 = -2a + 6$

$\lim_{x \downarrow a} f'(x) = \lim_{x \downarrow a} 4 = 4$

dus $-2a + 6 = 4$

$-2a = -2$

$a = 1$

dus f_a is differentieerbaar in $x = a$ als $a = 1$

Opgave 22:

a. $t(x) = x^3 - 8$

$t'(x) = 3x^2$

$t'(2) = 12$

raaklijn in $(2,0)$ is: $y = 12(x-2)$

$n(x) = x^2 - 4$

$n'(x) = 2x$

$n'(2) = 4$

raaklijn in $(2,0)$ is: $y = 4(x-2)$

b. $\lim_{x \to 2} \frac{x^3 - 8}{x^2 - 4} = \lim_{x \to 2} \frac{(x-2)(x^2 + 2x + 4)}{(x-2)(x+2)} = \lim_{x \to 2} \frac{x^2 + 2x + 4}{x + 2} = \frac{4 + 4 + 4}{4} = 3$

$\lim_{x \to 2} \frac{12(x-2)}{4(x-2)} = \lim_{x \to 2} \frac{12}{4} = 3$

dus $\lim_{x \to 2} \frac{x^3 - 8}{x^2 - 4} = \lim_{x \to 2} \frac{12(x-2)}{4(x-2)}$

$\lim_{x \to 2} \frac{t'(x)}{n'(x)} = \lim_{x \to 2} \frac{3x^2}{2x} = \lim_{x \to 2} \tfrac{3}{2} x = 3$

Opgave 23:

a. $\lim\limits_{x\to 3}\dfrac{\ln(x-2)}{x^2-9}=\lim\limits_{x\to 3}\dfrac{\frac{1}{x-2}}{2x}=\dfrac{1}{6}$

b. $\lim\limits_{x\to 3}\dfrac{x^3-27}{x^2-9}=\lim\limits_{x\to 3}\dfrac{3x^2}{2x}=\dfrac{27}{6}=4\tfrac{1}{2}$

c. $\lim\limits_{x\to 1}\dfrac{2x^3-x-1}{2(x^3-1)}=\lim\limits_{x\to 1}\dfrac{6x^2-1}{6x^2}=\dfrac{5}{6}$

d. $\lim\limits_{x\to 0}\dfrac{e^{2x}-1}{x}=\lim\limits_{x\to 0}\dfrac{2e^{2x}}{1}=\dfrac{2}{1}=2$

Opgave 24:

a. $\lim\limits_{x\to 2}\dfrac{2x^3-3x^2-12x+20}{3x^2-12x+12}=\lim\limits_{x\to 2}\dfrac{6x^2-6x-12}{6x-12}=\lim\limits_{x\to 2}\dfrac{12x-6}{6}=\dfrac{18}{6}=3$

b. $\lim\limits_{x\to 0}\dfrac{4x}{\sqrt{5-x}-\sqrt{5}}=\lim\limits_{x\to 0}\dfrac{4}{\frac{-1}{2\sqrt{5-x}}}=\lim\limits_{x\to 0}-8\sqrt{5-x}=-8\sqrt{5}$

c. $\lim\limits_{x\to 1}\dfrac{1-\sqrt{x}}{2-\sqrt{x+3}}=\lim\limits_{x\to 1}\dfrac{\frac{-1}{2\sqrt{x}}}{\frac{-1}{2\sqrt{x+3}}}=\lim\limits_{x\to 1}\dfrac{\sqrt{x+3}}{\sqrt{x}}=\dfrac{2}{1}=2$

d. $\lim\limits_{x\to 4}\dfrac{2^x-16}{x^2-16}=\lim\limits_{x\to 4}\dfrac{2^x\cdot\ln 2}{2x}=\dfrac{16\ln 2}{8}=2\ln 2$

16.2 Limieten en asymptoten

Opgave 25:

a. $f(-10) = 3{,}462$

$f(-100) = 3{,}059$

$f(-1000) = 3{,}006$

b. door x heel erg negatief te worden daalt $f(x)$ steeds dichter naar 3

c. $f(10) = 2{,}308$

$f(100) = 2{,}939$

$f(1000) = 2{,}994$

d. door x heel erg positief te nemen stijgt $f(x)$ steeds dichter naar 3

e. $y = 3$

Opgave 26:

a. $\lim\limits_{x\to\infty} \dfrac{3x^2 - x}{2 - x^2} = \lim\limits_{x\to\infty} \dfrac{3 - \frac{1}{x}}{\frac{2}{x^2} - 1} = \dfrac{3-0}{0-1} = -3$

b. $\lim\limits_{x\to\infty} \dfrac{5 - x^3}{2x^3} = \lim\limits_{x\to\infty} \dfrac{\frac{5}{x^3} - 1}{2} = \dfrac{0-1}{2} = -\frac{1}{2}$

c. $\lim\limits_{x\to\infty} \dfrac{6x^2}{2 - x^3} = \lim\limits_{x\to\infty} \dfrac{\frac{6}{x}}{\frac{2}{x^3} - 1} = \dfrac{0}{0-1} = 0$

d. $\lim\limits_{x\to\infty} \dfrac{(2x-3)^2}{x^2+1} = \lim\limits_{x\to\infty} \dfrac{4x^2 - 12x + 9}{x^2+1} = \lim\limits_{x\to\infty} \dfrac{4 - \frac{12}{x} + \frac{9}{x^2}}{1 + \frac{1}{x^2}} = \dfrac{4-0+0}{1+0} = 4$

Opgave 27:

a. $\lim\limits_{x\to\infty} \dfrac{(4x-1)^2}{x^3+4} = \lim\limits_{x\to\infty} \dfrac{16x^2 - 8x + 1}{x^3+4} = \lim\limits_{x\to\infty} \dfrac{\frac{16}{x} - \frac{8}{x^2} + \frac{1}{x^3}}{1 + \frac{4}{x^3}} = \dfrac{0-0+0}{1+0} = 0$

b. $\lim\limits_{x\to-\infty} \dfrac{x^3+2}{(3x+1)^2} = \lim\limits_{x\to-\infty} \dfrac{x^3+2}{9x^2+6x+1} = \lim\limits_{x\to-\infty} \dfrac{x + \frac{2}{x^2}}{9 + \frac{6}{x} + \frac{1}{x^2}} =$ bestaat niet

c. $\lim\limits_{x\to-\infty} \dfrac{4x}{\sqrt{x^2+1}} = \lim\limits_{x\to-\infty} \dfrac{4x}{\sqrt{x^2(1+\frac{1}{x^2})}} = \lim\limits_{x\to-\infty} \dfrac{4x}{-x\sqrt{1+\frac{1}{x^2}}} = \lim\limits_{x\to-\infty} \dfrac{-4}{\sqrt{1+\frac{1}{x^2}}} = \dfrac{-4}{\sqrt{1+0}} = -4$

d. $\sqrt{2-x}$ bestaat voor $x \le 2$ dus $\lim\limits_{x\to\infty} \dfrac{\sqrt{2-x}}{\sqrt{x+2}}$ bestaat niet

Opgave 28:

a. $\lim\limits_{x\to\infty} \dfrac{(2x+1)^3}{x^3+1} = \lim\limits_{x\to\infty} \dfrac{8x^3 + 12x^2 + 6x + 1}{x^3+1} = \lim\limits_{x\to\infty} \dfrac{8 + \frac{12}{x} + \frac{6}{x^2} + \frac{1}{x^3}}{1 + \frac{1}{x^3}} = \dfrac{8+0+0+0}{1+0} = 8$

b. $\lim\limits_{x\to\infty} \dfrac{\sqrt{5x} + 50}{x-1} = \lim\limits_{x\to\infty} \dfrac{\frac{1}{x} \cdot \sqrt{5x} + \frac{50}{x}}{1 - \frac{1}{x}} = \lim\limits_{x\to\infty} \dfrac{\sqrt{\frac{5}{x}} + \frac{50}{x}}{1 - \frac{1}{x}} = \dfrac{0+0}{1-0} = 0$

c. $\lim\limits_{x\to\infty} \dfrac{2\sqrt{x^2+1}}{x + \sqrt{x}} = \lim\limits_{x\to\infty} \dfrac{\frac{2}{x}\sqrt{x^2+1}}{1 + \frac{1}{x}\sqrt{x}} = \lim\limits_{x\to\infty} \dfrac{2\sqrt{1+\frac{1}{x^2}}}{1+\sqrt{\frac{1}{x}}} = \dfrac{2\sqrt{1+0}}{1+0} = 2$

d. $\lim\limits_{x\to-\infty}\dfrac{3x^2+x\sqrt{x^2+4}}{x^2+1}=\lim\limits_{x\to-\infty}\dfrac{3x^2+\sqrt{x^2(1+\frac{4}{x^2})}}{x^2+1}=\lim\limits_{x\to-\infty}\dfrac{3x^2-x^2\sqrt{1+\frac{4}{x^2}}}{x^2+1}=$

$$\lim_{x\to-\infty}\frac{3-\sqrt{1+\frac{4}{x^2}}}{1+\frac{1}{x^2}}=\frac{3-\sqrt{1+0}}{1+0}=2$$

Opgave 29:

$\lim\limits_{x\to\infty}\dfrac{4x^2-1}{x^2-x-6}=\lim\limits_{x\to\infty}\dfrac{4-\frac{1}{x^2}}{1-\frac{1}{x}-\frac{6}{x^2}}=\dfrac{4-0}{1-0-0}=4$ dus de H.A. is $y=4$

$x^2-x-6=0$

$(x-3)(x+2)=0$

$x=3 \quad \vee \quad x=-2$ dus de V.A. zijn $x=3$ en $x=-2$

Opgave 30:

$\lim\limits_{x\to\infty}\dfrac{6x}{\sqrt{2x^2-6}}=\lim\limits_{x\to\infty}\dfrac{6x}{\sqrt{x^2(2-\frac{6}{x^2})}}=\lim\limits_{x\to\infty}\dfrac{6x}{x\sqrt{2-\frac{6}{x^2}}}=\lim\limits_{x\to\infty}\dfrac{6}{\sqrt{2-\frac{6}{x^2}}}=\dfrac{6}{\sqrt{2+0}}=3\sqrt{2}$

$\lim\limits_{x\to-\infty}\dfrac{6x}{\sqrt{2x^2-6}}=\lim\limits_{x\to-\infty}\dfrac{6x}{\sqrt{x^2(2-\frac{6}{x^2})}}=\lim\limits_{x\to-\infty}\dfrac{6x}{-x\sqrt{2-\frac{6}{x^2}}}=\lim\limits_{x\to-\infty}\dfrac{6}{-\sqrt{2-\frac{6}{x^2}}}=\dfrac{6}{-\sqrt{2+0}}=-3\sqrt{2}$

dus de H.A. zijn $y=3\sqrt{2}$ en $y=-3\sqrt{2}$

$2x^2-6=0$

$2x^2=6$

$x^2=3$

$x=\sqrt{3} \quad \vee \quad x=-\sqrt{3}$ dus de V.A. zijn $x=\sqrt{3}$ en $x=-\sqrt{3}$

Opgave 31:

a. $bx^2-18=0$ voor $x=3$ en voor $x=-3$

$9b-18=0$

$9b=18$

$b=2$

$\lim\limits_{x\to\infty}\dfrac{ax^2+5}{2x^2-18}=\lim\limits_{x\to\infty}\dfrac{a+\frac{5}{x^2}}{2-\frac{18}{x^2}}=\dfrac{a+0}{2-0}=\frac{1}{2}a=6$

dus $a=12$

b. $ax^4+bx^3-2=0$ voor $x=-2$ en voor $x=2$

$$\begin{cases}16a-8b-2=0\\16a+8b-2=0 \quad +\end{cases}$$

$32a-4=0$

$32a=4$

$a=\frac{1}{8}$

$b=0$

Opgave 32:

a. $\lim_{x \to \infty} f(x) = 0$

b. $\lim_{x \to \infty} f(x) = \frac{a_n}{b_m}$

c. $\lim_{x \to \infty} f(x) =$ bestaat niet

Opgave 33:

a. $\frac{x^2+5x+2}{x+2} = \frac{x^2+5x+6-4}{x+2} = \frac{(x+2)(x+3)-4}{x+2} = x+3-\frac{4}{x+2}$

b. $\lim_{x \to \infty} \frac{4}{x+2} = \lim_{x \to \infty} \frac{\frac{4}{x}}{1+\frac{2}{x}} = \frac{0}{1+0} = 0$

$\lim_{x \to -\infty} \frac{4}{x+2} = \lim_{x \to -\infty} \frac{\frac{4}{x}}{1+\frac{2}{x}} = \frac{0}{1+0} = 0$

dus de grafiek gaat op den duur steeds meer lijken op de lijn $y = x+3$

Opgave 34:

a. $x-1=0$

$x=1$ dus de V.A. is $x=1$

$f(x) = \frac{2x^2+4x-5}{x-1} = 2x+6+\frac{1}{x-1}$ dus de S.A. is $y=2x+6$

b. $2x-1=0$

$2x=1$

$x=\frac{1}{2}$ dus de V.A. is $x=\frac{1}{2}$

$g(x) = \frac{-x^2+3x-2}{2x-1} = -\frac{1}{2}x+1\frac{1}{4}-\frac{\frac{3}{4}}{2x-1}$ dus de S.A. is $y=-\frac{1}{2}x+1\frac{1}{4}$

c. $x^2-9=0$

$x^2=9$

$x=3 \vee x=-3$ dus de V.A. zijn $x=3$ en $x=-3$

$h(x) = \frac{x^3+3x^2-9x+31}{x^2-9} = x+3+\frac{58}{x^2-9}$ dus de S.A. is $y=x+3$

d. $x^2-1=0$

$x^2=1$

$x=1 \vee x=-1$ dus de V.A. zijn $x=1$ en $x=-1$

$j(x) = \frac{x^3-1}{x^2-1} = x+\frac{x-1}{x^2-1}$ dus de S.A. is $y=x$

Opgave 35:

$f(x) = \frac{x^2+3x+2}{|x|} = \frac{x^2+3x+2}{x} = x+3+\frac{2}{x}$ als $x>0$

$f(x) = \frac{x^2+3x+2}{|x|} = \frac{x^2+3x+2}{-x} = -x-3-\frac{2}{x}$ als $x<0$

V.A.: $x=0$

S.A.: $y = x + 3$ als $x \to \infty$
$y = -x - 3$ als $x \to -\infty$

Opgave 36:

a. $f(x) = \frac{x^2 - x - 2}{x + 2} = \frac{(x+2)(x-3)+4}{x+2} = x - 3 + \frac{4}{x+2}$

V.A.: $x = -2$

S.A.: $y = x - 3$

b. $f'(x) = \frac{(x+2)(2x-1) - (x^2 - x - 2)}{(x+2)^2} = \frac{2x^2 + 3x - 2 - x^2 + x + 2}{(x+2)^2} = \frac{x^2 + 4x}{(x+2)^2} = 0$

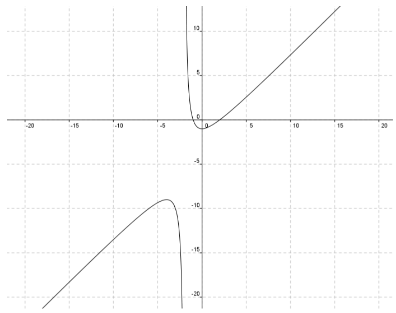
$x^2 + 4x = 0$

$x(x+4) = 0$

$x = 0 \quad \vee \quad x = -4$

max $f(-4) = -9$

min $f(0) = -1$



c. $x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2 \quad \vee \quad x = -1$

$$Opp(V) = -\int_{-1}^{2} \frac{x^2 - x - 2}{x+2}\,dx = -\int_{-1}^{2} (x - 3 + \frac{4}{x+2})\,dx =$$

$$= -\left[\tfrac{1}{2}x^2 - 3x + 4\ln|x+2|\right]_{-1}^{2}$$

$$= -(2 - 6 + 4\ln 4 - (\tfrac{1}{2} + 3 + 0)) = -(4\ln 4 - 7\tfrac{1}{2}) = 7\tfrac{1}{2} - 4\ln 4$$

Opgave 37:

a. $f(10) = \sqrt{340} = 18{,}44$

$f(100) = \sqrt{39400} = 198{,}5$

$f(1000) = \sqrt{3994000} = 1998{,}5$

$g(10) = 20$

$g(100) = 200$

$g(1000) = 2000$

b. als $x \to \infty$ dan $f(x) - g(x) \to 0$

c. als $x \to -\infty$ dan $f(x) - h(x) \to 0$

Opgave 38:

$f(x) = \sqrt{4x^2 - 6x}$

$f'(x) = \frac{4x - 3}{\sqrt{4x^2 - 6x}}$

$$a = \lim_{x\to\infty} \frac{4x-3}{\sqrt{4x^2 - 6x}} = \lim_{x\to\infty} \frac{4x-3}{\sqrt{x^2(4 - \frac{6}{x})}} = \lim_{x\to\infty} \frac{4x-3}{x\sqrt{4 - \frac{6}{x}}} = \lim_{x\to\infty} \frac{4 - \frac{3}{x}}{\sqrt{4 - \frac{6}{x}}} = \frac{4-0}{\sqrt{4-0}} = \frac{4}{2} = 2$$

$$b = \lim_{x\to\infty} \sqrt{4x^2 - 6x} - 2x = \lim_{x\to\infty} (\sqrt{4x^2 - 6x}) - 2x) \cdot \frac{\sqrt{4x^2 - 6x} + 2x}{\sqrt{4x^2 - 6x} + 2x} =$$

$$= \lim_{x\to\infty} \frac{4x^2-6x-4x^2}{\sqrt{4x^2-6x}+2x} = \lim_{x\to\infty} \frac{-6x}{\sqrt{4x^2(1-\frac{6}{4x})}+2x} = \lim_{x\to\infty} \frac{-6x}{2x\sqrt{1-\frac{3}{2x}}+2x} =$$

$$= \lim_{x\to\infty} \frac{-6}{2\sqrt{1-\frac{3}{2x}}+2} = \frac{-6}{2\sqrt{1-0}+2} = -1\tfrac{1}{2}$$

dus de S.A. is $y = 2x - 1\tfrac{1}{2}$

Opgave 39:

$f(x) = \sqrt{4x^2+6}$

$$f'(x) = \frac{1}{2\sqrt{4x^2+6}} \cdot 8x = \frac{4x}{\sqrt{4x^2+6}}$$

$$a = \lim_{x\to\infty} \frac{4x}{\sqrt{4x^2+6}} = \lim_{x\to\infty} \frac{4x}{\sqrt{x^2(4+\frac{6}{x^2})}} = \lim_{x\to\infty} \frac{4x}{x\sqrt{4+\frac{6}{x^2}}} = \lim_{x\to\infty} \frac{4}{\sqrt{4+\frac{6}{x^2}}} = \frac{4}{\sqrt{4+0}} = 2$$

$$b = \lim_{x\to\infty} \sqrt{4x^2+6} - 2x = \lim_{x\to\infty} (\sqrt{4x^2+6} - 2x) \cdot \frac{\sqrt{4x^2+6}+2x}{\sqrt{4x^2+6}+2x}$$

$$= \lim_{x\to\infty} \frac{4x^2+6-4x^2}{\sqrt{4x^2+6}+2x} = \lim_{x\to\infty} \frac{6}{\sqrt{4x^2(1+\frac{6}{4x^2})}+2x} = \lim_{x\to\infty} \frac{6}{2x\sqrt{1+\frac{6}{2x^2}}+2x}$$

$$= \lim_{x\to\infty} \frac{\frac{6}{x}}{2\sqrt{1+\frac{6}{2x^2}}+2} = \frac{0}{2\sqrt{1+0}+2} = 0$$

Dus als $x \to \infty$ dan is de S.A.: $y = 2x$

$$a = \lim_{x\to-\infty} \frac{4x}{\sqrt{4x^2+6}} = \lim_{x\to-\infty} \frac{4x}{\sqrt{x^2(4+\frac{6}{x^2})}} = \lim_{x\to-\infty} \frac{4x}{-x\sqrt{4+\frac{6}{x^2}}} = \lim_{x\to-\infty} \frac{4}{-\sqrt{4+\frac{6}{x^2}}} = \frac{-4}{\sqrt{4+0}} = -2$$

$$b = \lim_{x\to-\infty} \sqrt{4x^2+6} + 2x = \lim_{x\to-\infty} (\sqrt{4x^2+6} + 2x) \cdot \frac{\sqrt{4x^2+6}-2x}{\sqrt{4x^2+6}-2x}$$

$$= \lim_{x\to-\infty} \frac{4x^2+6-4x^2}{\sqrt{4x^2+6}-2x} = \lim_{x\to-\infty} \frac{6}{\sqrt{4x^2(1+\frac{6}{4x^2})}-2x} = \lim_{x\to-\infty} \frac{6}{-2x\sqrt{1+\frac{6}{2x^2}}-2x}$$

$$= \lim_{x\to\infty} \frac{\frac{6}{x}}{-2\sqrt{1+\frac{6}{2x^2}}-2} = \frac{0}{-2\sqrt{1+0}-2} = 0$$

Dus als $x \to -\infty$ dan is de S.A.: $y = -2x$

Opgave 40:

a. $f(x) = \sqrt{x^2-4x}$

$x^2 - 4x \geq 0$

$x(x-4) = 0$

$x = 0 \quad \vee \quad x = 4$

dus $x \leq 0 \quad \vee \quad x \geq 4$

b. $$f'(x) = \frac{1}{2\sqrt{x^2-4x}} \cdot (2x-4) = \frac{x-2}{\sqrt{x^2-4x}}$$

$$a=\lim_{x\to\infty}\frac{x-2}{\sqrt{x^2-4x}}=\lim_{x\to\infty}\frac{x-2}{\sqrt{x^2(1-\frac{4}{x})}}=\lim_{x\to\infty}\frac{x-2}{x\sqrt{1-\frac{4}{x}}}=\lim_{x\to\infty}\frac{1-\frac{2}{x}}{\sqrt{1-\frac{4}{x}}}=\frac{1-0}{\sqrt{1-0}}=1$$

$$b=\lim_{x\to\infty}\sqrt{x^2-4x}-x=\lim_{x\to\infty}(\sqrt{x^2-4x}-x)\cdot\frac{\sqrt{x^2-4x}+x}{\sqrt{x^2-4x}+x}$$

$$=\lim_{x\to\infty}\frac{x^2-4x-x^2}{\sqrt{x^2-4x}+x}=\lim_{x\to\infty}\frac{-4x}{\sqrt{x^2(1-\frac{4}{x})}+x}=\lim_{x\to\infty}\frac{-4x}{x\sqrt{1-\frac{4}{x}}+x}$$

$$=\lim_{x\to\infty}\frac{-4}{\sqrt{1-\frac{4}{x}}+1}=\frac{-4}{\sqrt{1-0}+1}=-2$$

dus voor $x\to\infty$ is de S.A. $y=x-2$

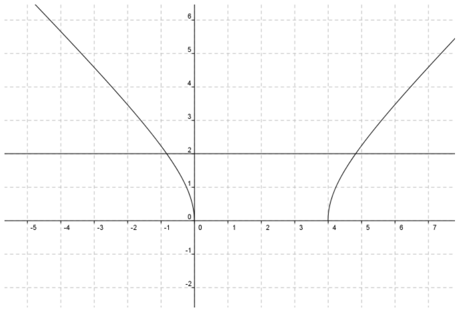
$$a=\lim_{x\to-\infty}\frac{x-2}{\sqrt{x^2-4x}}=\lim_{x\to-\infty}\frac{x-2}{\sqrt{x^2(1-\frac{4}{x})}}=\lim_{x\to-\infty}\frac{x-2}{-x\sqrt{1-\frac{4}{x}}}=\lim_{x\to-\infty}\frac{1-\frac{2}{x}}{-\sqrt{1-\frac{4}{x}}}=\frac{1-0}{-\sqrt{1-0}}=-1$$

$$b=\lim_{x\to-\infty}\sqrt{x^2-4x}+x=\lim_{x\to-\infty}(\sqrt{x^2-4x}+x)\cdot\frac{\sqrt{x^2-4x}-x}{\sqrt{x^2-4x}-x}$$

$$=\lim_{x\to-\infty}\frac{x^2-4x-x^2}{\sqrt{x^2-4x}-x}=\lim_{x\to-\infty}\frac{-4x}{\sqrt{x^2(1-\frac{4}{x})}-x}=\lim_{x\to-\infty}\frac{-4x}{-x\sqrt{1-\frac{4}{x}}-x}$$

$$=\lim_{x\to-\infty}\frac{-4}{-\sqrt{1-\frac{4}{x}}-1}=\frac{-4}{-\sqrt{1-0}-1}=2$$

dus voor $x\to-\infty$ is de S.A. $y=-x+2$



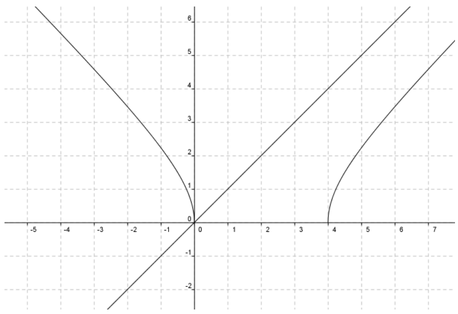
c. $\sqrt{x^2-4x}<2$

$x^2-4x=4$

$x^2-4x-4=0$

$x=\dfrac{4\pm\sqrt{32}}{2}=\dfrac{4\pm4\sqrt{2}}{2}=2\pm2\sqrt{2}$

$2-2\sqrt{2}<x\le 0 \quad \vee \quad 4\le x<2+2\sqrt{2}$



d. $\sqrt{x^2-4x}\le x$

$x^2-4x=x^2$

$-4x=0$

$x=0$

$x=0 \quad \vee \quad x\ge 4$

e. $Inh=\pi\int_4^p(\sqrt{x^2-4x})^2\,dx$

$$=\pi\int_4^p(x^2-4x)dx=\pi\left[\tfrac{1}{3}x^3-2x^2\right]_4^p=\pi\left[\tfrac{1}{3}p^3-2p^2-(21\tfrac{1}{3}-32)\right]=100\pi$$

$\frac{1}{3}p^3-2p^2+10\frac{1}{3}=100$

Deze vergelijking is niet met de hand op te lossen, wel met de GR.

Opgave 41:

a. $f(x)=\frac{1}{2}x-2+\dfrac{2}{x+1}$

$f'(x)=\frac{1}{2}-\dfrac{2}{(x+1)^2}=0$

$\dfrac{2}{(x+1)^2}=\frac{1}{2}$

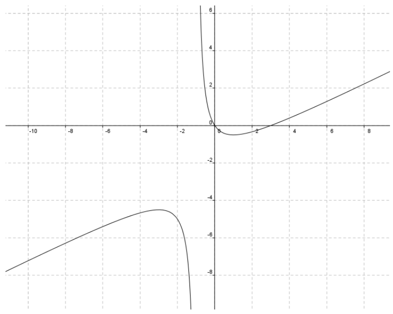
$(x+1)^2=4$

$x+1=2 \quad\vee\quad x+1=-2$

$x=1 \quad\vee\quad x=-3$

max $f(-3)=-4\frac{1}{2}$

min $f(1)=-\frac{1}{2}$



b. V.A.: $x=-1$

S.A.: $y=\frac{1}{2}x-2$

c. $\frac{1}{2}x-2+\dfrac{2}{x+1}=0$

$(\frac{1}{2}x-2)(x+1)+2=0$

$\frac{1}{2}x^2-1\frac{1}{2}x-2+2=0$

$\frac{1}{2}x(x-3)=0$

$x=0 \quad\vee\quad x=3$

$Opp(V)=-\int_0^3(\frac{1}{2}x-2+\dfrac{2}{x+1})dx=-\left[\frac{1}{4}x^2-2x+2\ln|x+1|\right]_0^3=-(2\frac{1}{4}-6+2\ln 4)$

$=3\frac{3}{4}-2\ln 4$

d. f moet continu zijn in $x=-3$ dus:

$\lim_{x\uparrow -3} f(x)=f(-3)=-4\frac{1}{2}$

$\lim_{x\downarrow -3} ax^3+bx^2+cx+d=-27a+9b-3c+d$

dus $-27a+9b-3c+d=-4\frac{1}{2}$

f moet differentieerbaar zijn in $x=-3$ dus moet ook nog gelden:

$\lim_{x\uparrow -3} f'(x)=\lim_{x\uparrow -3}\frac{1}{2}-\dfrac{2}{(x+1)^2}=\frac{1}{2}-\frac{1}{2}=0$

$\lim_{x\downarrow -3} f'(x)=\lim_{x\downarrow -3}3ax^2+2bx+c=27a-6b+c$

dus $27a-6b+c=0$

f moet continu zijn in $x=0$ dus:

$\lim_{x\uparrow 0} ax^3+bx^2+cx+d=d$

$\lim_{x\downarrow 0} f(x)=f(0)=0$

dus $d=0$

f moet differentieerbaar zijn in $x=0$ dus moet ook nog gelden:

$\lim_{x\uparrow 0} f'(x)=\lim_{x\uparrow 0}3ax^2+2bx+x=c$

$\lim_{x\downarrow 0} f'(x)=\lim_{x\downarrow 0}\frac{1}{2}-\dfrac{2}{(x+1)^2}=\frac{1}{2}-2=-1\frac{1}{2}$

dus $c=-1\frac{1}{2}$

$$\begin{cases} -27a+9b+4\frac{1}{2}=-4\frac{1}{2} \\ 27a-6b-1\frac{1}{2}=0 \quad + \end{cases}$$

$3b+3=-4\frac{1}{2}$

$3b=-7\frac{1}{2}$

$b=-2\frac{1}{2}$

$27a-6\cdot -2\frac{1}{2}-1\frac{1}{2}=0$

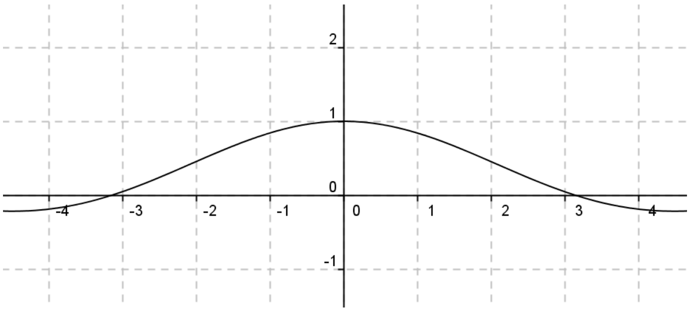
$27a=-13\frac{1}{2}$

$a=-\frac{1}{2}$

16.3 Standaardlimieten

Opgave 42:

a.



b. $\lim\limits_{x\to 0}\dfrac{\sin x}{x}=1$

Opgave 43:

a. $\sin\alpha=\dfrac{d(B,OA)}{1}=d(B,OA)$

$\tan\alpha=\dfrac{AC}{OA}=AC$

oppervlakte sector $OAB=\dfrac{\alpha}{2\pi}\cdot\pi r^2=\frac{1}{2}\alpha\cdot 1^2=\frac{1}{2}\alpha$

$Opp(\Delta OAB)=\frac{1}{2}\cdot OA\cdot d(B,OA)=\frac{1}{2}\cdot 1\cdot\sin\alpha=\frac{1}{2}\sin\alpha$

$Opp(\Delta OAC)=\frac{1}{2}\cdot OA\cdot AC=\frac{1}{2}\cdot 1\cdot\tan\alpha=\frac{1}{2}\tan\alpha$

b. $Opp(\Delta OAB)\le opp(\text{sec}\,tor\ OAB)\le Opp(\Delta OAC)$

$\frac{1}{2}\sin\alpha\le\frac{1}{2}\alpha\le\frac{1}{2}\tan\alpha$

$\sin\alpha\le\alpha\le\tan\alpha$

Opgave 44:

a. $\lim\limits_{x\uparrow 0}\dfrac{\sin x}{x}=\lim\limits_{-x\downarrow 0}\dfrac{\sin x}{x}=\lim\limits_{y\downarrow 0}\dfrac{\sin(-y)}{-y}=\lim\limits_{y\downarrow 0}\dfrac{-\sin y}{-y}=\lim\limits_{y\downarrow 0}\dfrac{\sin y}{y}=1$

b. $\lim\limits_{x\to 0}\dfrac{\tan x}{x}=\lim\limits_{x\to 0}\dfrac{\frac{\sin x}{\cos x}}{x}=\lim\limits_{x\to 0}\dfrac{\sin x}{x}\cdot\cos x=1\cdot 1=1$

Opgave 45:

a. $\lim\limits_{x\to 0}\dfrac{\sin 2x}{x}=\lim\limits_{x\to 0}\dfrac{\sin 2x}{2x}\cdot 2=1\cdot 2=2$

b. $\lim\limits_{x\to 0}\dfrac{x}{\sin 3x}=\lim\limits_{x\to 0}\dfrac{3x}{\sin 3x}\cdot\frac{1}{3}=1\cdot\frac{1}{3}=\frac{1}{3}$

c. $\lim\limits_{x\to 0}\dfrac{x^2+\tan x}{x}=\lim\limits_{x\to 0}(x+\dfrac{\tan x}{x})=0+1=1$

d. $\lim\limits_{x\to 0}\dfrac{\sin 4x}{\tan\frac{1}{2}x}=\lim\limits_{x\to 0}\dfrac{\sin 4x}{4x}\cdot\dfrac{\frac{1}{2}x}{\tan\frac{1}{2}x}\cdot\dfrac{4}{\frac{1}{2}}=1\cdot 1\cdot 8=8$

e. $\lim\limits_{x\to 0}\dfrac{2x+\sin x}{3x+\tan x}=\lim\limits_{x\to 0}=\lim\limits_{x\to 0}\dfrac{2+\frac{\sin x}{x}}{3+\frac{\tan x}{x}}=\dfrac{2+1}{3+1}=\frac{3}{4}$

f. $\lim\limits_{x\to\pi}\dfrac{2x\sin x}{\tan x}=\lim\limits_{x\to\pi}\dfrac{2x\sin x}{\frac{\sin x}{\cos x}}=\lim\limits_{x\to\pi}2x\cos x=2\pi\cdot -1=-2\pi$

Opgave 46:

a. $\lim\limits_{x\to\frac{1}{2}\pi}\dfrac{\sin(x-\frac{1}{2}\pi)}{x-\frac{1}{2}\pi}=\lim\limits_{y\to 0}\dfrac{\sin y}{y}=1$

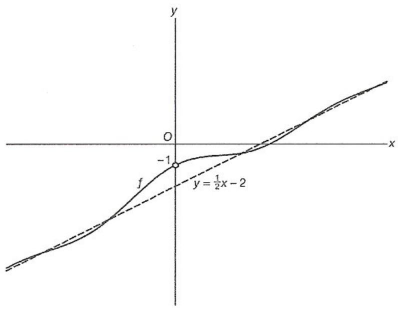
b. $\lim\limits_{x\to\frac{1}{2}\pi}\dfrac{\tan(x-\frac{1}{2}\pi)}{\sin(\frac{1}{2}x-\frac{1}{4}\pi)}=\lim\limits_{x\to\frac{1}{2}\pi}\dfrac{\tan(x-\frac{1}{2}\pi)}{\sin(\frac{1}{2}(x-\frac{1}{2}\pi))}=\lim\limits_{y\to 0}\dfrac{\tan y}{\sin\frac{1}{2}y}=\lim\limits_{y\to 0}\dfrac{\tan y}{y}\cdot\dfrac{\frac{1}{2}y}{\sin\frac{1}{2}y}\cdot 2=1\cdot 1\cdot 2=2$

c. $\lim\limits_{x\to 0}\dfrac{x^2+\sin 2x}{2x+\sin x}=\lim\limits_{x\to 0}\dfrac{2x+2\cos 2x}{2+\cos x}=\dfrac{0+2}{2+1}=\frac{2}{3}$ (gebruik de l'Hospital)

d. $\lim\limits_{x\downarrow 0}\dfrac{x\sqrt{x}+\tan x}{x+\sin 3x}=\lim\limits_{x\downarrow 0}\dfrac{1\frac{1}{2}\sqrt{x}+\frac{1}{\cos^w x}}{1+3\cos 3x}=\dfrac{0+1}{1+3}=\frac{1}{4}$ (gebruik de l'Hospital)

Opgave 47:

a. $\lim\limits_{x\to 0}\dfrac{x^2-4x+2\sin x}{2x}=\lim\limits_{x\to 0}(\frac{1}{2}x-2+\dfrac{\sin x}{x})=0-2+1=-1$



als $x\to\infty$ dan $f(x)\to\infty$

als $x\to-\infty$ dan $f(x)\to-\infty$

$B_f=\langle\leftarrow,-1\rangle\cup\langle -1,\rightarrow\rangle$

b. $y=\frac{1}{2}x-2$

c. $y_1=\dfrac{x^2-4x+2\sin x}{2x}$ optie zero geeft: $x=4{,}43$

dus $x\le 4{,}43$

Opgave 48:

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}=$$

$$=\lim_{h\to 0}\frac{2\sin(\frac{1}{2}(x+h-x))\cdot\cos(\frac{1}{2}(x+h+x))}{h}$$

$$=\lim_{h\to 0}\frac{2\sin\frac{1}{2}h\cdot\cos(x+\frac{1}{2}h)}{h}$$

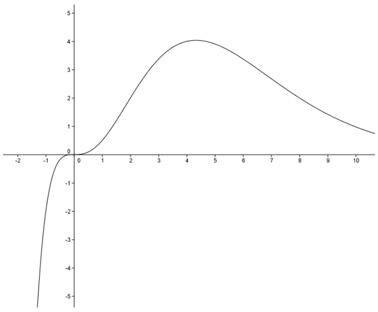
$$=\lim_{h\to 0}\frac{\sin\frac{1}{2}h\cdot\cos(x+\frac{1}{2}h)}{\frac{1}{2}h}$$

$$=\lim_{h\to 0}\frac{\sin\frac{1}{2}h}{\frac{1}{2}h}\cdot\cos(x+\tfrac{1}{2}h)$$

$$=1\cdot\cos x=\cos x$$

Opgave 49:

a.



b. $\lim\limits_{x\to\infty}\dfrac{x^3}{2^x}=0$

c. $f(x)=\dfrac{x^3}{2^x}=x^3\cdot 2^{-x}=2^{\,^2\log x^3}\cdot 2^{-x}=2^{3\cdot\,^2\log x}\cdot 2^{-x}=2^{3\cdot\,^2\log x-x}$

Opgave 50:

a. $\lim\limits_{x\to\infty}\dfrac{\ln x}{x}=\lim\limits_{t\to\infty}\dfrac{\ln e^{\frac{t}{n}}}{e^t}=\lim\limits_{t\to\infty}\dfrac{\frac{t}{n}}{e^t}=\lim\limits_{t\to\infty}\dfrac{1}{n}\cdot\dfrac{t}{e^t}=\dfrac{1}{n}\cdot\lim\limits_{t\to\infty}\dfrac{t}{e^t}=\dfrac{1}{n}\cdot 0=0$

b. $\lim\limits_{x\downarrow 0}(x^n\cdot\ln x)=\lim\limits_{t\to\infty}(\tfrac{1}{t})^n\cdot\ln\tfrac{1}{t}=\lim\limits_{t\to\infty}\dfrac{\ln t^{-1}}{t^n}=\lim\limits_{t\to\infty}\dfrac{-\ln t}{t^n}=0$

Opgave 51:

a. $\lim\limits_{x\to\infty}\dfrac{x\sqrt{x}}{e^{2x}}=\lim\limits_{x\to\infty}\dfrac{x^{1\frac{1}{2}}}{(e^2)^x}=0$

b. $\lim\limits_{x\to-\infty}x\cdot e^x=\lim\limits_{t\to\infty}-t\cdot e^{-t}=\lim\limits_{t\to\infty}-\dfrac{t}{e^t}=0$

c. $\lim\limits_{x\to\infty}\dfrac{\ln x}{x^2\sqrt{x}}=\lim\limits_{x\to\infty}\dfrac{\ln x}{x^{2\frac{1}{2}}}=0$

d. $\lim\limits_{x\to\infty}\dfrac{\ln^3 x}{\sqrt[3]{x}}=\lim\limits_{x\to\infty}\dfrac{\ln^3 x}{x^{\frac{1}{3}}}=\lim\limits_{x\to\infty}\left(\dfrac{\ln x}{x^{\frac{1}{9}}}\right)^3=0^3=0$

e. $\lim\limits_{x\downarrow 0}\sqrt[3]{x}\cdot\ln x=\lim\limits_{x\downarrow 0}x^{\frac{1}{3}}\cdot\ln x=0$

f. $\lim\limits_{x\downarrow 0}\sqrt{x}\cdot\ln^3 x=\lim\limits_{x\downarrow 0}x^{\frac{1}{2}}\cdot\ln^3 x=\lim\limits_{x\downarrow 0}(x^{\frac{1}{6}}\cdot\ln x)^3=0^3=0$

Opgave 52:

a. $\lim\limits_{x\to\infty}\dfrac{x^2+4x}{e^x}=\lim\limits_{x\to\infty}\left(\dfrac{x^2}{e^x}+4\cdot\dfrac{x}{e^x}\right)=0+4\cdot 0=0$

b. $\lim\limits_{x\to-\infty}x^4\cdot\sqrt{e^x}=\lim\limits_{t\to\infty}t^4\cdot e^{-\frac{1}{2}t}=\lim\limits_{t\to\infty}\dfrac{t^4}{e^{\frac{1}{2}t}}=\lim\limits_{t\to\infty}\dfrac{t^4}{(e^{\frac{1}{2}})^t}=0$

c. $\lim\limits_{x\to\infty}\dfrac{\sqrt[3]{x}}{x^e}=\lim\limits_{x\to\infty}\dfrac{\ln^{\frac{1}{3}}x}{x^e}=\lim\limits_{x\to\infty}\left(\dfrac{\ln x}{x^{3e}}\right)^{\frac{1}{3}}=0^{\frac{1}{3}}=0$

d. $\lim\limits_{x\to\infty}\left(\frac{10}{x}\right)^e \cdot \sqrt{\ln x} = \lim\limits_{x\to\infty}\frac{10^e}{x^e}\cdot \ln^{\frac{1}{2}} x = 10^e \cdot \lim\limits_{x\to\infty}\left(\frac{\ln x}{x^{2e}}\right)^{\frac{1}{2}} = 10^e \cdot 0 = 0$

e. $\lim\limits_{x\downarrow 0}(x^2+x)\ln x = \lim\limits_{x\downarrow 0}(x^2\ln x + x\ln x) = 0+0=0$

f. $\lim\limits_{x\downarrow 0} x^2 e^x \ln^3 x = \lim\limits_{x\downarrow 0} e^x \cdot (x^{\frac{2}{3}}\ln x)^3 = e^0 \cdot 0^3 = 1\cdot 0 = 0$

Opgave 53:

a. $\lim\limits_{x\downarrow 0} x^x = \lim\limits_{x\downarrow 0} e^{\ln x^x} = \lim\limits_{x\downarrow 0} e^{x\cdot\ln x} = e^0 = 1$

b. $\lim\limits_{x\downarrow 0} x^{\frac{1}{x}} = \lim\limits_{x\downarrow 0} e^{\ln x^{\frac{1}{x}}} = \lim\limits_{x\downarrow 0} e^{\frac{1}{x}\cdot\ln x} = 0 \qquad (e^{-\infty} = 0)$

c. $\lim\limits_{x\to\infty} x^{\frac{1}{x}} = \lim\limits_{x\to\infty} e^{\ln x^{\frac{1}{x}}} = \lim\limits_{x\to\infty} e^{\frac{\ln x}{x}} = e^0 = 1$

Opgave 54:

a. $f(x) = ex\cdot\ln^2 x$

$f'(x) = e\cdot\ln^2 x + ex\cdot 2\ln x\cdot\frac{1}{x} = e\cdot\ln^2 x + 2e\cdot\ln x = 0$

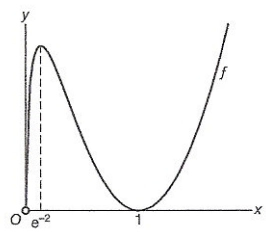
$e\cdot\ln x\cdot(\ln x + 2) = 0$

$\ln x = 0 \quad\vee\quad \ln x = -2$

$x = e^0 = 1 \quad\vee\quad x = e^{-2} = \frac{1}{e^2}$

max $f(\frac{1}{e^2}) = \frac{4}{e}$

min $f(1) = 0$



b. $\lim\limits_{x\downarrow 0} ex\cdot\ln^2 x = e\cdot\lim\limits_{x\downarrow 0}(x^{\frac{1}{2}}\ln x)^2 = e\cdot 0^2 = 0$

c. $y_1 = ex\cdot\ln^2 x$ en $y_2 = \ln x$

intersect geeft $x = 1 \quad\vee\quad x = 1{,}32$

$0 < x < 1 \quad\vee\quad x > 1{,}32$

Opgave 55:

a. $f(x) = x^{\frac{1}{x}} = e^{\ln x^{\frac{1}{x}}} == e^{\frac{1}{x}\ln x}$ f

$f'(x) = e^{\frac{1}{x}\ln x}\cdot(\frac{-1}{x^2}\ln x + \frac{1}{x}\cdot\frac{1}{x}) = e^{\frac{1}{x}\ln x}\cdot\frac{1}{x^2}\cdot(-\ln x + 1) = 0$

$e^{\frac{1}{x}\ln x} = 0 \quad\vee\quad \frac{1}{x^2} = 0 \quad\vee\quad -\ln x + 1 = 0$

$-\ln x = -1$

$\ln x = 1$

$x = e$

max $f(e) = e^{\frac{1}{e}}$

b. $\lim\limits_{x\downarrow 0} f(x) = \lim\limits_{x\downarrow 0} x^{\frac{1}{x}} = \lim\limits_{x\downarrow 0}(e^{\ln x})^{\frac{1}{x}} = \lim\limits_{x\downarrow 0} e^{\frac{\ln x}{x}} = 0$

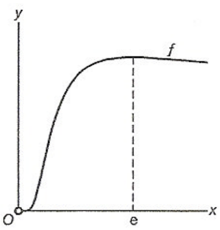
$f'(x) = \frac{1}{x^2}\cdot e^{\frac{1}{x}\ln x}\cdot(1-\ln x) = \frac{x^{\frac{1}{x}}}{x^2}(1-\ln x) = x^{\frac{1}{x}-2}\cdot(1-\ln x) = x^{\frac{1}{x}-2} - x^{\frac{1}{x}-2}\cdot\ln x$

neem $p = \frac{1}{x} - 2$ dan is $\frac{1}{x} = p + 2$ dus $x = \frac{1}{p+2}$

$$\lim_{x\downarrow 0} x^{\frac{1}{x}-2} = \lim_{p\to\infty}\left(\tfrac{1}{p+2}\right)^p = 0$$

$$\lim_{x\downarrow 0} x^{\frac{1}{x}-2}\cdot \ln x = 0 \text{ (standaardlimiet)}$$

dus $\lim_{x\downarrow 0} f'(x) = 0 - 0 = 0$



c. $\lim_{x\to\infty} x^{\frac{1}{x}} = \lim_{x\to\infty} e^{\frac{\ln x}{x}} = e^0 = 1$ dus de H.A. is $y = 1$

d. $f(x) = g(x)$

$x^{\frac{1}{x}} = x^x$

$\frac{1}{x} = x$

$x^2 = 1$

$x = 1 \quad \vee \quad x = -1$ (vervalt)

$f'(1) = \frac{1}{1^2}\cdot e^{\frac{1}{1}\cdot\ln 1}\cdot(1-\ln 1) = 1$

$g'(1) = 1^1 = 1$

$f(1) = g(1) \quad \wedge \quad f'(1) = g'(1)$ dus de grafieken van *f* en *g* raken elkaar in $x = 1$

Opgave 56:

a. $f(x) = e^x$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{e^{x+h}-e^x}{h}$$

b. $\lim_{h\to 0}\frac{e^{x+h}-e^x}{h} = \lim_{x\to 0}\frac{e^x(e^h-1)}{h} = e^x\cdot\lim_{h\to 0}\frac{e^h-1}{h}$

$f'(x) = e^x$ dus $\lim_{h\to 0}\frac{e^x-1}{h} = 1$

Opgave 57:

$$\lim_{x\to-\infty}\left(1+\tfrac{a}{x}\right)^x = \lim_{t\to\infty}\left(1+\tfrac{a}{-t}\right)^{-t} = \lim_{t\to\infty}\frac{1}{\left(1+\frac{-a}{t}\right)^t} = \frac{1}{e^{-a}} = e^a$$

Opgave 58:

a. $\lim_{x\to\infty}\left(1-\tfrac{2}{x}\right)^x = \lim_{x\to\infty}\left(1+\tfrac{-2}{x}\right)^x = e^{-2} = \tfrac{1}{e^2}$

b. $\lim_{x\to\infty}\left(\tfrac{x+4}{x}\right)^x = \lim_{x\to\infty}\left(1+\tfrac{4}{x}\right)^x = e^4$

c. $\lim_{x\to\infty}\left(\tfrac{x}{x+5}\right)^x = \lim_{x\to\infty}\left(\frac{1}{1+\frac{5}{x}}\right)^x = \lim_{x\to\infty}\frac{1^x}{\left(1+\frac{5}{x}\right)^x} = \frac{1}{e^5}$

d. $\lim_{x\to-\infty}\left(\tfrac{2x+1}{2x}\right)^x = \lim_{x\to-\infty}\left(1+\tfrac{1}{2x}\right)^x = \lim_{x\to\infty}\left(1+\frac{\frac{1}{2}}{x}\right)^x = e^{\frac{1}{2}} = \sqrt{e}$

Opgave 59:

a. stel $x^2 = t$

$\lim_{x\to\infty}\left(1+\tfrac{1}{x^2}\right)^{x^2} = \lim_{t\to\infty}\left(1+\tfrac{1}{t}\right)^t = e^1 = e$

b. $\lim_{x\to\infty}\left(\tfrac{x^2+4}{x^2}\right)^{x^2} = \lim_{x\to\infty}\left(1+\tfrac{4}{x^2}\right)^{x^2} = e^4$

$$\lim_{x\to\infty}\left(\frac{x^2+4}{x^2}\right)^x = \lim_{x\to\infty}\left(1+\frac{4}{x^2}\right)^x = \lim_{x\to\infty}\left(\left(1+\frac{4}{x^2}\right)^{x^2}\right)^{\frac{1}{x}} = (e^4)^0 = 1$$

16.4 Limieten bij rijen

Opgave 60:

a. $S_{10} = \frac{1}{2}\cdot 11\cdot(u_0 + u_{10}) = \frac{1}{2}\cdot 11\cdot(5+35) = 220$

b. $S_{19} = \frac{1}{2}\cdot 20\cdot(u_0 + u_{19}) = \frac{1}{2}\cdot 20\cdot(5+65) = 670$

c. $S_n = \frac{1}{2}(n+1)(u_0 + u_n) = \frac{1}{2}(n+1)(5+5+3n)$

$= (\frac{1}{2}n + \frac{1}{2})(10+3n)$

$= 5n + 1\frac{1}{2}n^2 + 5 + 1\frac{1}{2}n$

$= 1\frac{1}{2}n^2 + 6\frac{1}{2}n + 5$

Opgave 61:

a. $S_{10} = \dfrac{u_0 - u_{11}}{1-r} = \dfrac{20 - 20\cdot(\frac{3}{2})^{11}}{1-\frac{3}{2}} = 3419{,}9$

b. $S_{19} = \dfrac{u_0 - u_{20}}{1-r} = \dfrac{20 - 20\cdot(\frac{3}{2})^{20}}{1-\frac{3}{2}} = 132970$

c. $S_n = \dfrac{u_0 - u_{n+1}}{1-r} = \dfrac{20 - 20\cdot(\frac{3}{2})^{n+1}}{1-\frac{3}{2}} = -40 + 40\cdot(\frac{3}{2})^{n+1} = -40 + 40\cdot\frac{3}{2}\cdot(\frac{3}{2})^n = 60\cdot(\frac{3}{2})^n - 40$

Opgave 62:

a. $S_{10} = \dfrac{u_0 - u_{11}}{1-r} = \dfrac{60 - 60\cdot(\frac{2}{3})^{11}}{1-\frac{2}{3}} = 177{,}9$

b. $S_{19} = \dfrac{u_0 - u_{20}}{1-r} = \dfrac{60 - 60\cdot(\frac{2}{3})^{20}}{1-\frac{2}{3}} = 179{,}95$

c. $\lim\limits_{n\to\infty} u_n = \lim\limits_{n\to\infty} 60\cdot(\frac{2}{3})^n = 0$

$\lim\limits_{n\to\infty}\sum\limits_{k=0}^{n} u_k = \lim\limits_{n\to\infty}\dfrac{u_0 - u_{n+1}}{1-r} = \lim\limits_{n\to\infty}\dfrac{60 - 60\cdot(\frac{2}{3})^{n+1}}{1-\frac{2}{3}} = \dfrac{60-0}{\frac{1}{3}} = 180$

Opgave 63:

a. $2+5+8+...+(3n+2) = \frac{1}{2}\cdot(n+1)\cdot(2+3n+2)$

$= \frac{1}{2}(n+1)(3n+4)$

$= \frac{1}{2}(3n^2+7n+4)$

$= 1\frac{1}{2}n^2 + 3\frac{1}{2}n + 2$

$\lim\limits_{n\to\infty}\dfrac{2+5+8+...+(3n+2)}{n^2} = \lim\limits_{n\to\infty}\dfrac{1\frac{1}{2}n^2 + 3\frac{1}{2}n + 2}{n^2} = \lim\limits_{n\to\infty}\dfrac{1\frac{1}{2} + \frac{3\frac{1}{2}}{n} + \frac{2}{n^2}}{1} = \dfrac{1\frac{1}{2}+0+0}{1} = 1\frac{1}{2}$

b. $1+2+3+...+n = \frac{1}{2}n(1+n) = \frac{1}{2}n^2 + \frac{1}{2}n$

$\lim\limits_{n\to\infty}\dfrac{1+2+3+...+n}{n^2} = \lim\limits_{n\to\infty}\dfrac{\frac{1}{2}n^2 + \frac{1}{2}n}{n^2} = \lim\limits_{n\to\infty}\dfrac{\frac{1}{2}+\frac{1}{2n}}{1} = \dfrac{\frac{1}{2}+0}{1} = \frac{1}{2}$

c. $1+2+4+8+...+2^n = \dfrac{1-2^{n-1}}{1-2} = 2^{n+1} - 1$

$$\lim_{n\to\infty}\frac{1+2+4+8+...+2^n}{2^n}=\lim_{n\to\infty}\frac{2^{n+1}-1}{2^n}=\lim_{n\to\infty}(2-2^{-n})=2-0=2$$

d. $10+20+40+...+5\cdot 2^n=\frac{10-5\cdot 2^{n+1}}{1-2}=5\cdot 2^{n+1}-10$

$$\lim_{n\to\infty}\frac{5\cdot 2^n}{10+20+40+...+5\cdot 2^n}=\lim_{n\to\infty}\frac{5\cdot 2^n}{5\cdot 2^{n+1}-10}=\lim_{n\to\infty}\frac{5}{10-\frac{10}{2^n}}=\frac{5}{10-0}=\tfrac{1}{2}$$

Opgave 64:

a. $1+5+9+13+...+4n+1=\tfrac{1}{2}(n+1)(1+4n+1)=(\tfrac{1}{2}n+\tfrac{1}{2})(4n+2)=2n^2+3n+1$

$1+4+7+10+...+3n+1=\tfrac{1}{2}(n+1)(1+3n+1)=(\tfrac{1}{2}+\tfrac{1}{2}n)(3n+2)=1\tfrac{1}{2}n^2+2\tfrac{1}{2}n+1$

$$\lim_{n\to\infty}\frac{1+5+9+13+...+4n+1}{1+4+7+10+...+3n+1}=\lim_{n\to\infty}\frac{2n^2+3n+1}{1\tfrac{1}{2}n^2+2\tfrac{1}{2}n+1}=\lim_{n\to\infty}\frac{2+\frac{3}{n}+\frac{1}{n^2}}{1\tfrac{1}{2}+\frac{2\frac{1}{2}}{n}+\frac{1}{n^2}}=\frac{2+0+0}{1\tfrac{1}{2}+0+0}=\tfrac{4}{3}$$

b. $$\lim_{n\to\infty}\frac{2+5+9+13+...+4n}{2+6+11+16+...+5n}=\lim_{n\to\infty}\frac{1+5+9+13+...+4n+1}{1+6+11+16+...+5n+1}$$

$$=\lim_{n\to\infty}\frac{\tfrac{1}{2}(n+1)(1+4n+1)}{\tfrac{1}{2}(n+1)(1+5n+1)}$$

$$=\lim_{n\to\infty}\frac{4n+2}{5n+2}=\lim_{n\to\infty}\frac{4+\frac{2}{n}}{5+\frac{2}{n}}=\frac{4+0}{5+0}=\tfrac{4}{5}$$

c. $$\lim_{n\to\infty}\frac{1-2+4-8+...+(-2)^n}{2-4+8-16+...+-1\cdot(-2)^{n+1}}=\lim_{n\to\infty}\frac{1-2+4-8+...+(-2)^n}{2\cdot(1-2+4-8+...+(-2)^n)}=\lim_{n\to\infty}\tfrac{1}{2}=\tfrac{1}{2}$$

d. $100+150+200+...+100+50n=\tfrac{1}{2}(n+1)(100+100+50n)$

$=(\tfrac{1}{2}n+\tfrac{1}{2})(200+50n)$

$=25n^2+125n+100$

$1+1{,}5+2{,}25+...+1{,}5^n=\frac{1-1{,}5^{n+1}}{1-1{,}5}=-2+2\cdot 1{,}5^{n+1}=-2+2\cdot 1{,}5^n\cdot 1{,}5=3\cdot 1{,}5^n-2$

$$\lim_{n\to\infty}\frac{100+150+200+...+100+50n}{1+1{,}5+2{,}25+...+1{,}5^n}=\lim_{n\to\infty}\frac{25n^2+125n+100}{3\cdot 2^n-2}$$

$$=\lim_{n\to\infty}\frac{\frac{25n^2}{2^n}+\frac{125n}{2^n}+\frac{100}{2^n}}{3-\frac{2}{2^n}}=\frac{0+0+0}{3-0}=0$$

Opgave 65:

a. $400+300+225+186{,}75+126{,}5625+...=\frac{400}{1-0{,}75}=1600$

b. $1-\tfrac{1}{2}+\tfrac{1}{4}-\tfrac{1}{8}+\tfrac{1}{16}-\tfrac{1}{32}+...=\frac{1}{1--\tfrac{1}{2}}=\tfrac{2}{3}$

c. $S=\frac{40}{1-0{,}9}=400$

d. $S=\frac{500}{1-\tfrac{2}{3}}=1500$

Opgave 66:

$u_n = 200 \cdot 0{,}8^n$

$S = \dfrac{200}{1-0{,}8} = 1000\,\text{cm}$

Opgave 67:

a. $135 + 2 \cdot 135 \cdot 0{,}7 + 135 \cdot 0{,}7^2 = 390{,}15\,\text{cm}$

b. omlaag geldt: $135 + 135 \cdot 0{,}7 + 135 \cdot 0{,}7^2 + 135 \cdot 0{,}7^3 + \ldots = \dfrac{135}{1-0{,}7} = 450$

omhoog geldt: $135 \cdot 0{,}7 + 135 \cdot 0{,}7^2 + 135 \cdot 0{,}7^3 + 135 \cdot 0{,}7^4 + \ldots = \dfrac{135 \cdot 0{,}7}{1-0{,}7} = 315$

dus totaal: $450 + 315 = 765\,\text{cm}$

Opgave 68:

omlaag geldt: $40 + 40 \cdot 0{,}4 \cdot 0{,}75 + 40 \cdot (0{,}4 \cdot 0{,}7)^2 + 40 \cdot (0{,}4 \cdot 0{,}75)^3 + \ldots = \dfrac{40}{1-0{,}4 \cdot 0{,}75} = 57\tfrac{1}{7}$

omhoog geldt: $40 \cdot 0{,}4 + 40 \cdot 04{,}\cdot 0{,}4 \cdot 0{,}75 + 40 \cdot 0{,}4 \cdot (0{,}4 \cdot 0{,}75)^2 + 40 \cdot 0{,}4 \cdot (0{,}4 \cdot 0{,}75)^3 + \ldots =$

$\dfrac{40 \cdot 0{,}4}{1-0{,}4 \cdot 0{,}75} = 22\tfrac{6}{7}$

dus totaal: $57\tfrac{1}{7} + 22\tfrac{6}{7} = 80\,\text{m}$

Opgave 69:

a. $h_0 = 6$

$h_1 = 3\sqrt{2}$

$r = \dfrac{3\sqrt{2}}{6} = \tfrac{1}{2}\sqrt{2}$

$h_n = 6 \cdot (\tfrac{1}{2}\sqrt{2})^n$

$S_7 = \dfrac{6 - 6 \cdot (\tfrac{1}{2}\sqrt{2})^8}{1 - \tfrac{1}{2}\sqrt{2}} = 19{,}2\,\text{cm}$

b. $S = \dfrac{6}{1-\tfrac{1}{2}\sqrt{2}} = 20{,}5\,\text{cm}$

c. iedere zijde wordt $\tfrac{1}{2}\sqrt{2}$ keer zo groot, dus de inhoud wordt $(\tfrac{1}{2}\sqrt{2})^3 = \tfrac{1}{4}\sqrt{2}$ keer zo groot

$I_n = 216 \cdot (\tfrac{1}{4}\sqrt{2})^n$

$S = \dfrac{216}{1-\tfrac{1}{4}\sqrt{2}} = 334{,}134\,\text{cm}^3$